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Supersymmetric non-polynomial vector multiplets and causal propagation

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Abstract. The infinite class of massless spin-1 actions formed from the two algebraic invariants $F_{\mu\nu}F^{\mu\nu}$, $F_{\mu\nu}{}^*F^{\mu\nu}$ which allow a supersymmetric extension is derived. It is shown that (to second nonlinear order at least) these extensions all have causal propagation, even though only one of them (Born–Infeld) was causal before supersymmetrisation.

1. Introduction

Supersymmetric theories have the remarkable property that they have positive energy (Iliopoulos and Zumino 1974) and consequently that they do not admit spacelike eigenvalues of the total four-momentum, i.e. they have no global tachyonic solutions (Deser 1979). Since non-polynomial theories with higher derivatives usually have causality problems, a natural question is whether it is possible to construct sypersymmetric non-polynomial Lagrangians, and whether these are locally causal. We investigate this for spin-1 actions, which we try to extend supersymmetrically, starting from the general non-derivative form which can depend only on the two algebraic Maxwell invariants, $\int \mathscr{L}_n(F^2, F^*F)$. In performing the construction, we expected the Born–Infeld theory to be the appropriate candidate for a supersymmetric extension, because this non-polynomial theory is the only massless spin-1 theory, aside from Maxwell's, which is causal and possesses a unique characteristic cone (Plebanski 1968). It turns out, however, that the Born–Infeld theory is but one of an infinite class of theories which allow supersymmetric extensions. Since the supersymmetry transformations are the usual order-preserving ones, $\delta A_\mu = \mathrm{i}\bar{\alpha}\gamma_\mu\psi$, $\delta\psi = -F(\sigma\alpha)$, functions at any polynomial order preserve the supersymmetry, and we can investigate propagation properties order by order. One can then show that, to sixth order in $F_{\mu\nu}$ at least, all these models remain causal by virtue of the interaction structure. Thus supersymmetry improves causality, even of those models whose bosonic ancestors were acausal.

2. Lagrangian construction

The general gauge-invariant Lagrangian L_F for a massless spin-1 field A_μ which depends on $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ (but not on explicit derivatives) is a function of the two algebraic invariants $X = F^{\mu\nu}F_{\mu\nu} \equiv FF$, $Y \equiv {}^*FF$, ${}^*F^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\sigma\tau}F_{\sigma\tau}$. We may expand it as

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follows:

$$L_F = f(X, Y^2)$$
$$= -\frac{X}{4} + \sum_{n=1}^{\infty} \sum_{\nu=0}^{n} (a_{2n-2\nu,2\nu} X^{2n-2\nu} + a_{2n-2\nu+1,2\nu} X^{2n-2\nu+1}) Y^{2\nu}. \tag{1}$$

The a coefficients are proportional to a dimensional coupling constant $g \sim L^4$ with

$$a_{n,m} \sim g^{n+m-1}.$$

We have restricted ourselves to parity invariant actions, although the following results are also valid in the general case. In particular the Born–Infeld $L_F^{(\mathrm{BI})}$ is given by

$$gf^{(\mathrm{BI})} = -[-\det(\eta_{\mu\nu} + g^{1/2} F_{\mu\nu})]^{1/2} + 1 = -(1 + \tfrac{1}{2} gX - \tfrac{1}{16} g^2 Y^2)^{1/2} + 1.$$

The coefficients are therefore, in units $g = 1$,

$$a^{(\mathrm{BI})}_{n-\nu,\nu} = (-1)^{n+\nu} \left(\frac{1}{2}\right)^{3n+6\nu-1} \frac{1}{n} \binom{2n-2}{n-1} \binom{n}{\nu}. \tag{2}$$

Our aim is to construct a supersymmetric extension of (1), that is a supersymmetric Lagrangian which contains L_F as its pure bosonic part. In order to achieve this we must introduce a fermionic partner for the field A_μ and an auxiliary field. The smallest supermultiplet containing the spin-1 field A_μ is

$$(A_\mu, \psi_\alpha, D)$$

where ψ_α is a Weyl spinor and the auxiliary field D is a pseudoscalar. The supertransformation is

$$\delta A_\mu = \mathrm{i}(\alpha\sigma_\mu\bar{\psi} - \psi\sigma_\mu\bar{\alpha}) \qquad \delta\psi = \sigma^{\mu\nu}\alpha F_{\mu\nu} + \mathrm{i}\alpha D \qquad \delta D = -\partial_\mu\psi\sigma^\mu\bar{\alpha} - \alpha\sigma^\mu\, \partial_\mu\bar{\psi}. \tag{3}$$

For our purposes we found it most convenient to use the superfield formalism. The corresponding superfield W_α whose components are (A_μ, ψ_α, D) is obtained by applying to the superfield

$$V(\theta, \bar{\theta}, x) = V^\dagger(\theta, \bar{\theta}, x)$$

the covariant derivative $\bar{D}.\bar{D}^{\cdot} D_\alpha$, that is

$$W_\alpha = \bar{D}_{\dot{\beta}} \bar{D}^{\dot{\beta}} D_\alpha V$$

with (Wess 1978 and references therein) $D_\alpha = \partial/\partial\theta^\alpha + \mathrm{i}\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu$, $\bar{D}_{\dot{\alpha}} = (D_\alpha)^\dagger$. In terms of W_α the free action invariant under (3) is

$$S_0 = \int \mathrm{d}^4x \left(\frac{1}{2}\right)^8 \left(\frac{\partial^2}{\partial\theta^2} W^2 \bigg|_{\theta=\bar{\theta}=0} + \mathrm{HC}\right)$$
$$= \int \mathrm{d}^4x [-\tfrac{1}{2}\mathrm{i}(\psi\sigma\,\partial\bar{\psi} - \mathrm{HC}) - \tfrac{1}{4}X + \tfrac{1}{2}D^2],$$

the usual $(1, \frac{1}{2})$ supermultiplet in terms of Weyl spinors. In order to construct L_F in a supersymmetric way, we generate higher powers of X and Y in the last component of a superfield by multiplying superfields which are derived from W_α. Because

$$\bar{D}_{\dot{\alpha}} W_\alpha = 0$$

one may also build

$$D_\alpha W_\beta \equiv W_{\alpha\beta}, \qquad D_\alpha D_\beta W_\gamma \equiv W_{\alpha\beta\gamma}.$$

It turns out that $W_{\alpha\beta\gamma}$ contains X and Y with derivatives only. This is also the case for invariants arising from $W^{\alpha\beta}W_{\alpha\beta}$ and its powers, although $W^{\cdot\cdot}W_{\cdot\cdot}$ is of importance for the construction as we shall see below. We are left with the field $W^2\bar{W}^2$ as the only possibility that gives rise to invariants which contain X^2 and Y^2. We explicitly write out the last component $D_{W^2\bar{W}^2}$ which is of interest here:

$$D_{W^2\bar{W}^2} = \hat{L}_F + \hat{L}_{F,\psi} + \hat{L}_\psi + \hat{L}_D \tag{4}$$

where

$$\hat{L}_F \equiv 2^6[X^2 + Y^2]$$

$$\hat{L}_{F,\psi} \equiv -2^8[{}^*F^{\mu\nu}\,\partial^\rho F_{\mu\rho}\psi\sigma_\nu\bar{\psi} - 2\mathrm{i}F^{\mu\nu}F_{\mu\rho}(\psi\sigma_\nu\,\partial^\rho\bar{\psi} - \mathrm{HC})$$
$$-\tfrac{1}{2}\mathrm{i}X(\psi\sigma\,\partial\bar{\psi} - \mathrm{HC}) + Y(\psi\sigma\,\partial\bar{\psi} + \mathrm{HC})]$$

$$\hat{L}_\psi \equiv 2^{10}(\psi\sigma\,\partial\bar{\psi})(\partial\psi\sigma\bar{\psi}) - \tfrac{1}{2}(\psi\partial_\mu\psi)(\bar{\psi}\,\partial^\mu\bar{\psi}) + \tfrac{1}{8}[\psi\psi\,\partial^\mu(\bar{\psi}\,\partial_\mu\bar{\psi}) + \mathrm{HC}]\}$$

$$\hat{L}_D \equiv -2^8\{-D^4 + D^2[X + 3\mathrm{i}(\psi\sigma\,\partial\bar{\psi} - \mathrm{HC})]$$
$$+2D(\partial^\mu F_{\nu\mu}\psi\sigma^\nu\bar{\psi} - {}^*F_{\mu\nu}(\psi\sigma^\mu\,\partial^\nu\bar{\psi} - \mathrm{HC}) - 2F^{\mu\nu}\,\partial_\nu D\psi\sigma_\mu\bar{\psi}\}.$$

So far we have constructed a nonlinear spin-1–spin-$\frac{1}{2}$ system up to first order in the coupling constant g. Because W_α is nilpotent it is not possible to produce higher-order terms in X and Y from powers of W_α. However, the field $W_{\alpha\beta}$ is not nilpotent, and forming

$$W^{\alpha\beta}W_{\alpha\beta}$$

we observe that this superfield has the first component $A_{W^{\cdot\cdot}W_{\cdot\cdot}}$ given by

$$\tfrac{1}{32}A_{W^{\cdot\cdot}W_{\cdot\cdot}} = D^2 - \tfrac{1}{2}X - \tfrac{1}{2}\mathrm{i}Y. \tag{5}$$

We mention that $W^{\cdot\cdot}W_{\cdot\cdot}$ and $\bar{W}_{\cdot\cdot}\bar{W}^{\cdot\cdot}$ are the only superfields (besides W^2 and $W^2\bar{W}^2$) derived from W_α which contain combinations of X and Y. From (5) we learn that the combined fields

$$W_F \equiv W^{\cdot\cdot}W_{\cdot\cdot} + \bar{W}_{\cdot\cdot}\bar{W}^{\cdot\cdot} \qquad \text{and} \qquad W_{*F} \equiv \mathrm{i}(W^{\cdot\cdot}W_{\cdot\cdot} - \bar{W}_{\cdot\cdot}\bar{W}^{\cdot\cdot})$$

have the first components

$$\tfrac{1}{64}A_{W_F} = D^2 - \tfrac{1}{2}X \qquad \tfrac{1}{64}A_{W_{*F}} = \tfrac{1}{2}Y. \tag{6}$$

It follows that

$$S_{n,m} = (W_F)^n(W_{*F})^m W^2\bar{W}^2$$

has (among other contributions) a term proportional to

$$X^nY^m(X^2 + Y^2)$$

in the last component. Hence it is possible to construct a supersymmetric extension of L_F given by (1) if we can find coefficients $b_{n,m}$ such that in a given order $g^{2m-1}(m \geq 1)$ of the coupling constant the relation

$$\sum_{\nu=0}^{m} a_{2m-2\nu,2\nu}X^{2m-2\nu}Y^{2\nu} = \sum_{\nu=0}^{m-1} b_{2m-2\nu-2,2\nu}X^{2m-2\nu-2}Y^{2\nu}(X^2 + Y^2) \tag{7}$$

together with the corresponding relation of order g^{n-1} holds. It immediately follows that this construction is only possible if the coefficients $a_{n-\nu,\nu}$ which determine the order g^{n-1} part of L_F obey the constraints

$$0=\sum_{\nu=0}^{m}(-1)^{\nu}a_{2m-2\nu,2\nu}\qquad 0=\sum_{\nu=0}^{m}(-1)^{\nu}a_{2m-2\nu+1,2\nu},\tag{8}$$

and in this case $b_{n,m}$ is

$$\begin{aligned}b_{2m-2n-2,2n}&=\sum_{\nu=0}^{n}(-1)^{\nu}a_{2m-2n+2\nu,2n-2\nu}\\ b_{2m-2n-1,2n}&=\sum_{\nu=0}^{n}(-1)^{\nu}a_{2m-2n+2\nu+1,2n-2\nu}.\end{aligned}\tag{9}$$

Note that the last term in (7), (X^2+Y^2), is the famous Euler–Heisenberg first nonlinear term in the Born–Infeld action, $\frac{1}{32}g^2(X^2+Y^2)$. Since the equations (8) are the only constraints which are imposed on L_F by supersymmetry, their solutions generate all Lagrangians L_F which possess a sypersymmetric extension in the sense stated above. These can be either polynomial or not, since the constraints only link a's of the same order. The corresponding supersymmetric Lagrangian L_S is

$$\begin{aligned}L_S=&\left(\frac{1}{2}\right)^{8}\left(\frac{\partial^2}{\partial\theta^2}W^2\bigg|_{\theta=\bar\theta=0}+\mathrm{HC}\right)+\left(\frac{1}{2}\right)^{15}\left(\frac{\partial^2}{\partial\theta^2}\frac{\partial^2}{\partial\bar\theta^2}W^2\bar W^2\right)\\ &+\left(\frac{1}{2}\right)^{-4}\sum_{m=2}^{\infty}\sum_{\nu=0}^{m-1}\left(\frac{1}{2}\right)^{10m-2\nu}b_{2m-2\nu-2,2\nu}L_{2m-2\nu-2,2\nu}\\ &-\frac{1}{2}\sum_{m=1}^{\infty}\sum_{\nu=0}^{m-1}\left(\frac{1}{2}\right)^{10m-2\nu}b_{2m-2\nu-1,2\nu}L_{2m-2\nu-1,2\nu}\end{aligned}\tag{10}$$

where

$$L_{n,m}\equiv\left(\frac{1}{2}\right)^{4}\frac{\partial^2}{\partial\theta^2}\frac{\partial^2}{\partial\bar\theta^2}S_{n,m}.$$

In the polynomial case, the sums are truncated at the desired order. Note, however, that we use the term 'polynomial' before elimination of the auxiliary field, D. The latter can be expressed as a non-rational root of X and $\bar\psi\psi$ combinations. It will of course be polynomial in $\bar\psi\psi$, but their coefficients will in general be non-polynomial in X, and so will the action after elimination of D.

3. Solving the constraints

Before giving the general solution of the constraints (8) we show that the Born–Infeld theory has a supersymmetric extension. Considering $f^{(\mathrm{BI})}(X, Y^2)$ as a generating function of the coefficients $a^{(\mathrm{BI})}_{n-\nu,\nu}$ given in (2), we set

$$Y^2=-X^2$$

and obtain

$$\begin{aligned}f^{(\mathrm{BI})}(X,-X^2)&=-\tfrac{1}{4}X\\ &=-\frac{X}{4}+\sum_{n=1}^{\infty}\sum_{\nu=0}^{n}\left[(-1)^{\nu}a^{(\mathrm{BI})}_{2n-2\nu,2\nu}X^{2n}+(-1)^{\nu}a^{(\mathrm{BI})}_{2n-2\nu+1,2\nu}X^{2n+1}\right]\end{aligned}$$

which proves the assertion that (8) is satisfied, since X is arbitrary. For the general solution of (8) we write $a_{n-\nu,\nu}$ in the form

$$a_{n-\nu,\nu}=\frac{1}{n!}\binom{n}{\nu}C_{n-\nu,\nu}, \qquad C_{n,m}=\frac{\partial^{n+m}}{\partial X^n\,\partial Y^m}f(X,Y^2)|_{X=Y=0}.$$

The constraints (8) become

$$0=\sum_{\nu=0}^{m}(-1)^\nu\binom{2m}{2\nu}C_{2m-2\nu,2\nu} \qquad 0=\sum_{\nu=0}^{m}(-1)^\nu\binom{2m+1}{2\nu}C_{2m-2\nu+1,2\nu}. \tag{11}$$

The equations (11) are linear homogeneous difference equations of order $2m$ which have the solutions

$$\begin{aligned} C_{2m-2\nu,2\nu} &= \sum_{K=1}^{m}\xi_K\left[\cot\frac{\pi}{2}\left(\frac{2K-1}{2m}\right)\right]^{2\nu} \\ C_{2m-2\nu+1,2\nu} &= \sum_{K=1}^{m}\zeta_K\left[\tan\frac{\pi}{2}\left(\frac{2K-1}{2m+1}\right)\right]^{2\nu}. \end{aligned} \tag{12}$$

ξ_K and ζ_K each form a set of m constants corresponding to orders g^{2m-1} and g^{2m} respectively. In particular we find that up to order g all L_F's coincide with the truncated Born–Infeld $L_F^{(\mathrm{BI})}$, because we may absorb an overall constant in g. At order g^2 we have coincidence of L_F and $L_F^{(\mathrm{BI})}$ with respect to the relative coefficients of the two cubic invariants X^3 and Y^2X. However, since there are no suitable boundary conditions to be imposed on the C's from the construction alone, (12) defines an infinite class of supersymmetric Lagrangians, that is, we are no longer able to identify all the theories with the truncation of Born–Infeld in higher orders, having used up the freedom of g-choice to get agreement to order g. Of course, only the Born–Infeld case has a purely bosonic part which is causal.

4. Causality

The result of our construction shows that the Born–Infeld theory is in fact not the only possible non-polynomial theory which allows a supersymmetric generalisation. Moreover, it is possible to truncate the Lagrangian (10) to any order without losing the global causal behaviour, guaranteed by the supersymmetry algebra with $\{\bar{Q},Q\}=2\gamma P$, since the transformation rules (3) are of homogeneous order in the fields. But the pure spin-1 part we started with no longer generates the principal part of the equations of motion, i.e. the highest derivatives terms which determine the propagation behaviour. In fact, if we look for instance at (4) which is the leading term in a supersymmetric Lagrangian up to first order in g, we see that the principal part is generated by interactions, which contain higher derivatives than the original principal part generated by (1).

How do these theories behave locally? Global causality does not guarantee that there is no locally acausal behaviour, since only integrated quantities enter in the global algebra. In order to check the local propagation character, we determine the characteristics of the equations of motion (Courant and Hilbert 1965), which can be done separately for any supersymmetric truncation of (10). Up to order g the principal part is given by the expression (4), which leads to second-order equations for F and ψ and to an algebraic constraint for D. Upon eliminating D by expanding it in powers of g and inserting it into the equation of F and ψ, we obtain up to the required order the following principal part:

$$
\begin{aligned}
M^{\mu\nu}_{(B)ij}\,\partial_\mu\,\partial_\nu B_j + N^{\mu\nu}_{ij}\,\partial_\mu\,\partial_\nu E_j + (P^{\mu\nu}_{(B)i}\,\partial_\mu\,\partial_\nu\psi + \text{HC}) &= j_i \\
M^{\mu\nu}_{(E)ij}\,\partial_\mu\,\partial_\nu E_j + (P^{\mu\nu}_{(E)i}\,\partial_\mu\,\partial_\nu\psi + \text{HC}) &= j_i \\
\phi^{\mu\nu}_{\dot\alpha\alpha}\,\partial_\mu\,\partial_\nu\psi^\alpha = \bar V_{\dot\alpha} \qquad \bar\phi^{\mu\nu}_{\alpha\dot\alpha}\,\partial_\mu\,\partial_\nu\bar\psi^{\dot\alpha} &= -V_\alpha
\end{aligned}
\tag{13}
$$

where

$$
M^{\mu\nu}_{(B)ij} = (\psi\sigma_0\bar\psi)\eta^{\mu\nu}\delta_{ij} \qquad M^{\mu\nu}_{(E)ij} = (\psi\psi\bar\psi\bar\psi)\eta^{\mu\nu}\delta_{ij}
$$

$$
\phi^{\mu\nu} = -\bar\psi\psi\eta^{\mu\nu} + (\psi\sigma^\mu)(\sigma^\nu\bar\psi)
$$

and N, $P_{(B)}$, $P_{(E)}$, j_B, j_E and $\bar V$ are functions of the fields F and ψ and their first-order derivatives. By writing $\vec B$, $\vec E$, ψ and $\bar\psi$ in terms of a 10×1 matrix and replacing ∂_μ by n_μ, the unit normal to the characteristic surface S to be determined, we obtain from (13) the characteristic determinant C:

$$
C(n_\mu) = \begin{vmatrix} \hat M_{(B)} & \hat N & \hat P_{(B)} & \hat P^*_{(B)} \\ 0 & \hat M_{(E)} & \hat P_{(E)} & \hat P^*_{(E)} \\ 0 & 0 & \hat\phi & 0 \\ 0 & 0 & 0 & \hat{\bar\phi} \end{vmatrix} = \det \hat M_{(B)} \det \hat M_{(E)} \det \hat\phi \det \hat{\bar\phi}.
$$

Here $\hat M \equiv M^{\mu\nu} n_\mu n_\nu$ etc. The roots of the polynomial

$$
C(n_\mu) = 0 \tag{14}
$$

then determine the characteristic surface. The factorised structure of C shows that the off-diagonal coefficients N, $P_{(B)}$ and $P_{(E)}$ do not have any influence on the roots of (14), which are separately determined by the highest derivative parts of the spin-1 and spin-$\frac{1}{2}$ field. It follows immediately that $\det \hat M_{(B,E)} \sim n^6$ and $\det \hat\phi \sim n^4$, that is the characteristics are the usual light cones. Thus, up to this order causality holds locally as well. In order g^2 the equations of motion for F and ψ are third-order equations and the field D up to this order is

$$
D = \tfrac{1}{2}g[\partial_\nu F^{\mu\nu}\psi\sigma_\mu\bar\psi + \tfrac{1}{2}F^{\mu\nu}(\partial_\nu\psi\sigma_\mu\bar\psi + \text{HC}) + \tfrac{1}{2}\mathrm{i}\,{}^*F^{\mu\nu}(\partial_\nu\psi\sigma_\mu\bar\psi - \text{HC})] + \mathrm{O}(g^2).
$$

Due to the first derivative of F in D there are third-derivative contributions in the spin-1 equation originating from appearance of $\partial_\mu\,\partial_\nu D$ terms in the equations at order g. Taking this into account, we end up with the principal part

$$
\begin{aligned}
&(\psi\psi\bar\psi\bar\psi)\Box\,\partial^\mu F_{\nu\mu} + (P_{\nu\rho\sigma\tau}\,\partial^\rho\,\partial^\sigma\,\partial^\tau\psi + \text{HC}) = j_\nu \\
&\mathrm{i}(\psi\psi\bar\psi\bar\psi)\Box\,\partial_\mu\psi\sigma^\mu = \bar V.
\end{aligned}
\tag{15}
$$

Again C has block form, i.e. the coefficients P do not affect causality, and we learn that also in this order the characteristics are light cones. However, in all higher orders the coefficient matrices of F and ψ in the principal parts cause technical problems, and we have not checked what happens there. We stress that switching off the ψ and D interactions leaves one with the truncated non-supersymmetric spin-1 part, which is acausal, because there only first derivatives of $F_{\mu\nu}$ enter and these are always acausal, except Born–Infeld (Plebanski 1968). This follows from the fact that although the characteristic determinant of Born–Infeld theory is of course causal order by order in $F_{\mu\nu}$, the reverse is false: one cannot truncate the action and then obtain a causal determinant. An alternative way to check local causality in higher orders is to keep the auxiliary field D as a dynamical field and add it to the system of equations for F and ψ by differentiating the second-order equation in which it naturally appears. The contribution of D to the principal part is then

$$Q^{\lambda\mu\nu\rho}\,\partial^0\,\partial_\lambda\,\partial_\mu F_{\nu\rho}+R^{\lambda\mu}\,\partial^0\,\partial_\lambda\,\partial_\mu D+(T^{\lambda\mu}\,\partial^0\,\partial_\lambda\,\partial_\mu\psi+\text{HC})=K.$$

In considering this system it turns out that the principal part of the ψ field always decouples from the F–D system and may therefore be calculated separately. At order g^2 F and D also decouple, i.e. $Q=0$, and the coefficients R in this case are given by

$$R^{\lambda\mu}=(\psi\psi\bar{\psi}\bar{\psi})\eta^{\lambda\mu}.$$

The relevant coefficients for F and ψ are the same as in (15). This shows causal propagation of the auxiliary field D and proves local causality for the g^2-truncated theory to all orders in g. In this case, if we switch off the ψ and D interactions, none of the theories considered remains causal.

5. Conclusion

We have learned that supersymmetry improves local causal behaviour by introducing higher-order interaction terms. Of course, not all such higher-order terms would lead to causal theories. For example, in order to generate a leading second-order derivative term in the equations of motion of the spin-1 field, we might have chosen to add the (parity preserving) term

$$Y\,\partial_\lambda F^{\mu\lambda}\psi\sigma_\mu\bar{\psi}$$

to the action. This is an acausal theory, as can be shown by applying the procedure described above. But it is also impossible to incorporate this term into a larger, supersymmetric, action, since the number of fields is odd, and cannot be expressed in terms of superfields. On the other hand if we restrict ourselves to terms in the action which have the same power of fields as in the supersymmetric choice (14), we find that the only possible second-order derivative term is the causal one picked by supersymmetry.

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